Academic Curriculum

There are four main areas covered in math competitions: Algebra, Combinatorics, Geometry, and Number Theory. Our goal at the AwesomeMath Summer Program is to build and hone students' problem solving skills in these four fields. We offer 12 courses. Each course lasts for the duration of the camp and covers one of the four areas mentioned above (though there will be some overlap between disciplines; many techniques are helpful for different kinds of problems.) Each course meets 10 times per week devoted to 5 lectures and 5 problem sessions taught by a group of one or two instructors and two or three mentors and assistants. We call such a group an **academic team**. The instructor(s) of each academic team will teach the 90-minute lecture, and the staff members will provide hands-on support in the 90-minute problem sessions following the lecture. Because communication plays an important role in developing problem solving skills, students will be asked to participate actively in the problem sessions, i.e. to ask questions and present and defend solutions. The students will each select two courses, one in the morning and one in the afternoon, based on the following criteria:

- Students' personal choices (please see below for course selection tips);
- Students' mathematical background and ability as reflected in the personal application, recommendation letters, and achievements on AMC8 (2006 and 2007), AMC10/12 (2006, 2007, 2008), AIME (2006,2007, 2008), ARML (2006 and 2007), USAMO, and AwesomeMath Summer Camp (2006 and 2007).

When you arrive, we will go over your selections with you to ensure that you are in the courses that suit you best. Switching between courses will be allowed only in exceptional cases and only during the first week. Permission to change will be granted based on the recommendations of the academic teams.

Academic courses

Focal areas	Morning lecture/Problem session	Afternoon lecture/Problem session 2
Algebra	Algebra 1.5	
		Algebra 2.5
	Algebra 3.5	
Combinatorics		Math counts
	Counting Strategies	
		Combinatorial Arguments
Geometry	Elements of Geometry	
		Computational Geometry
	Geometric Proofs	
Number Theory		Number Sense
	Modular Arithmetic	
		Number Theory

Course selection tips

- Pick the areas in which you are the most interested, or on which you need the most work.
- The content of each course is very important. Please read the course description carefully.
- Every single one of our courses is very challenging and by far beyond the scope of regular/accelerated/honor classes in school settings. There is an **entry level** course in each area. Each of these entry level course will be challenging for most able young minds because these are **contest mathematics** courses.

Course description

Algebra courses

- *Algebra 1.5* Develops essential skills such as factoring, grouping, recognizing roots. Solving (systems of) equations/inequalities (linear, absolute value, quadratic, rational, radical) is the main theme of the course. Discriminants, Viète's relations, and symmetric polynomials also play a central role. **This is the entry level algebra course. It covers all AMC levels and easy end of AIME and ARML.** Here are four sample questions:

Sample 1. Which of the numbers 33, 42, 54, 56, 58, 62, and 73 is the average of the other six numbers? (Hint: One only needs to compute exactly one average.)

Sample 2. [AHSME 1998, by Richard Parris] In an h-meter race, Sunny is exactly d meters ahead of Windy when Sunny finishes the race. The next time they race, Sunny sportingly starts d meters behind Windy, who is at the starting line. Both runners run at the same constant speed as they did in the first race. How many meters ahead is Sunny when Sunny finishes the second race?

Sample 3. [AHSME 1999] The graph of $y = -|x - a| + b$ and $y = |x - c| + d$ intersects at points $(2, 5)$ and $(8, 3)$. Find $a + c$. (Hint: One can solve this in one step, if he notice a parallelogram.)

Sample 4. [ARML 2005] Find all real numbers a such that the equation

$$x^3 - ax^2 - 2ax + a^2 - 1 = 0$$

has exactly one real solution in x. (Hint: This is a quadratic equation.)

- *Algebra 2.5* Studies special (systems) of equations, discriminants, Viète's relations, symmetric polynomials, functional properties. Introduces (weighted) AM-GM-HM and Cauchy-Scharz inequalities. **This is the intermediate level algebra course. It covers the hard end of AMC12, and the medium to hard end of ARML and AIME. A student qualified for AIME with a score of 8 or below should be a good fit for this course.**

- *Algebra 3.5* Discusses functional equations, classical inequalities such as AM-GM-HM, Cauchy-Schwarz, Power-mean, and Jensen's inequalities, as well as Miruhead's and Schur's inequalities, and inequalities related to symmetric polynomials. **This is the advanced level algebra course. It covers the hard end of AIME and all levels of USAMO. A student with an AIME score of 8 or above should consider this course.**

Combinatorics courses

- *Math Counts* Studies the Addition or Multiplication Principles, permutations and combinations, and probability. Learns how to deal with over-counting and many useful properties of integer divisors. **This is the entry level combinatorics course. It covers MATHCOUNTS, all the AMC levels and the easy end of AIME and ARML.** Here are four sample questions:

Sample 5. [AHSME 1990] An $11 \times 11 \times 11$ wooden cube is formed by gluing together 11^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point? (Hint: The answer is the difference of two cubic numbers.)

Sample 6. [AHSME 1992] Ten points are selected on the positive x-axis, $\mathbf{X}^+$, and five points are selected on the positive y-axis, $\mathbf{Y}^+$. The fifty segments connecting the ten points on $\mathbf{X}^+$ to the five points on $\mathbf{Y}^+$ are drawn. What is the maximum possible number of points of intersection of these fifty segments in the interior of the first quadrant? (Hint: Count the number of quadrilaterals instead.)

Sample 7. [HMMT 2003] You are given a 10×2 grid of unit squares. Two different squares are adjacent if they share a side. How many ways can one mark exactly nine of the squares so that no two marked squares are adjacent? (Hint: There is a column of unmarked squares.)

Sample 8. [ARML 2005] One tries to pick three distinct numbers from the set $\{1, 2, \ldots, 34\}$ such that the sum of the three number is divisible by 3. In how many ways can this be done? (Hint: The answer is not 2005, but close.)

- *Counting Strategies* Discusses counting strategies such as the Addition or Multiplication Principles, permutations and combinations, properties of the binomial coefficients, bijections, recursions, and the Inclusion-Exclusion principle. **This is the intermediate level combinatorics course. It covers the hard end of AMC12, the medium to hard end of AIME and ARML, as well as the beginning USAMO level.**

- *Combinatorics* Introduces methods of mathematical proofs – induction, proofs by contradiction, the Pigeonhole Principle, the well-ordering principle, colorings, assigning weights, bijections/mappings, recursions, calculating in two ways, and combinatorial constructions. A focal point of the course is Combinatorial Number Theory. **This is the advanced level combinatorics course. It covers the hard end of AIME and the medium to hard end of USAMO.**

Geometry courses

- *Elements of Geometry* Deals with computational geometry elements (angles, side lengths, ratio, areas, etc.) in triangles, quadrilaterals (parallelograms, kites, trapezoids), and polygons (equilateral, equiangular, regular) as well as recognizing congruences and similarities. Analytic geometry (lines, circles, vectors, planes) is also a focal point. **This is the entry level geometry course. It covers MATHCOUNTS, all AMC levels, and the easy end of AIME and ARML.**Please do not under estimate the phrase *entry level*. Here are four sample questions:

Sample 9. [AHSME 1999] The equiangular convex hexagon $ABCDEF$ has $AB = 1$, $BC = 4$, $CD = 2$, and $DE = 4$. Find the area of the hexagon. (Hint: Complete the hexagon to a equilateral triangle.)

Sample 10. [MathCounts 2007, by Chengde Feng] Points A_1, B_1, C_1, D_1 lie on sides AB, BC, CD, DA of the unit square $ABCD$ with $AB = 2A_1B = 2B_1C = 2C_1D = 2D_1A$. What is the area of the square enclosed by segments AC_1, BD_1, CA_1, DB_1?

Sample 11. [AHSME 1996] In the coordinate plane, what is the length of the shortest path from $(0, 0)$ to $(12, 16)$ that does not go inside the circle $(x - 6)^2 + (y - 8)^2 = 25$? (Hint: The path consists of three pieces.)

Sample 12. [AMC 2006] A circle of radius r is concentric with and outside of a regular hexagon of side length 2. The probability that three entire sides of the hexagon are visible from a randomly chosen point on the circle is $\frac{1}{2}$. What is r?

- *Computational Geometry* Studies non-synthetic techniques in solving geometry problems: coordinate geometry, vectors (2-D and 3-D), planes, spheres, trigonometry, and complex numbers. Features many important geometric themes: The Law of Sines and the Law of Cosines, Ptolemy's theorem, Ceva's and Menelaus's theorems, Stewart's theorem, Heron's and Brahmagupta's formulas, Brocard points, dot product and the vector form of the Law of Cosines, the Cauchy-Schwarz inequality, 3-D coordinate systems, as well as linear representation and traveling on the earth (sphere). **This is the intermediate level geometry course. It covers the hard end of AMC12, the medium to hard end of AIME and ARML. A student with a AIME score of 4 to 8 should consider this course.**

- *Geometry Proofs* Focuses on classical topics such as concurrency, collinearity, cyclic quadrilaterals, special centers/points of triangles, and geometric constructions. Introduces important transformations – translation, reflections, and spiral similarities, with a touch on projective and inversive geometry. **This is the advanced level geometry course. It covers the hard end of AIME and the medium to hard end of USAMO. A student who is familiar with geometry and with a AIME score of 8 or above should consider this course.**

Number Theory courses

- *Number Sense* Studies divisibility, factoring, numerical systems, divisors and arithmetic functions of divisors. Setting-up and solving linear Diophantine equations is also a focal point of the course. **This is the entry level number theory course. It covers MATHCOUNTS, all AMC levels, and easy end of AIME and ARML.** Here are four sample questions:

Sample 13. [AHSME 1999] Four girls – Mary, Alina, Tina, and Hanna – sang songs in a concert as trios, with one girl sitting out each time. Hanna sang 7 songs, which was more than any other girls, and Mary sang 4 songs, which is fewer than any other girl. How many songs did these trios sing?

Sample 14. [AHSME 1998] In the sixth, seventh, eighth, and ninth basketball games of the season, a player scored 23, 14, 11, and 20 points, respectively. Her points-per-game average was higher after nine games than it was after the first five games. If her average after ten games was greater than 18, what is the least number of points she could have scored in the tenth game? (Hint: The answer is not 28.)

Sample 15. [AMC 2004] In the following 3×3 array of positive integers, the products of the entries of each row, column, and diagonal are the same. What is the sum of all the possible values of g?

$$\begin{array}{ccc} 50 & b & c \\ d & e & f \\ g & h & 2 \end{array}$$

(Hint: Write each term in terms of g.)

Sample 16. Four positive integers a, b, c, and d satisfy

$$\begin{aligned} ab + a + b &= 524, \\ bc + b + c &= 146, \\ cd + c + d &= 104. \end{aligned}$$

Find all the possible values of $a - d$. (Hint: Factoring.)

- *Modular Arithmetic* Develops essential skills in number theory: divisibility, the division algorithm, prime numbers, the Fundamental Theorem of Arithmetic, G.C.D., L.C.M., Bézout's identity, Euclidean algorithm, modular arithmetic, divisibility criteria in the decimal system. Studies numerical functions such as the number of divisors or the sum of divisors of integers. **This is the intermediate level number theory course. It covers the hard end of AMC12, the medium to hard end of AIME and ARML. A student with an AIME score of 4 to 10 should consider this course.**

- *Number Theory* Focuses on in-depth discussions about Diophantine equations, residue classes, quadratic reciprocity, Fermat's little theorem, Euler's theorem, primitive roots, and Euler's totient function, etc. **This is the advanced level number theory course. It covers the hard end of AIME and the medium to hard end of USAMO. A student who is familiar with number theory and with an AIME score of 10 or above should consider this course.**

Teaching materials and study tools

Teaching materials – hand-outs, problem sets, paper – will be provided to students.

A Typical Day at AwesomeMath

During the camp, there will be two no-class Mondays, July 21 and 28, for special activities and trips. Tuesday through Saturday classes will follow a schedule close to the one below. The academic load for weekends will be adjusted accordingly. Academic team contests will be held on Sunday evenings.

8:00 AM - 9:00 AM	Morning routine and breakfast
9:00 AM - 10:30 AM	Lectures
10:45 AM - 12:15 AM	Problem sessions
12:15 PM - 2:00 PM	Lunch break
2:00 PM - 3:30 PM	Lectures
3:45 PM - 5:15 PM	Problem sessions
5:15 PM - 7:00 PM	Dinner
7:00 PM - 8:30 PM	Mathematics Forum (optional)
7:00 PM - 10:00 PM	Recreational activities (optional)
10:00 PM - 11:00 PM	Check-in
11:00 PM	Lights out