



**AwesomeMath**  
making x,y,z as easy as a,b,c

## AwesomeMath Admission Test Cover Sheet

Your Name

Last Name

First Name

Admission Test

A

B

C

Check one

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Number of pages (not including this cover sheet)

# Admission Test B

Mar.28 - Apr.7, 2008

1. Construct a square using tiles of the form



2. Find all primes  $p$  such that  $47p^2 + 1$  is a perfect square.
3. The numbers 1 through 10 are written on a board. By deleting some of them, the remaining numbers can be divided into two groups such that the product of the numbers in each group is the same. What is the least number of numbers that need to be deleted?
4. Find the least positive integer  $n$  such that for each prime  $p$ ,  $p^2 + n$  is not a prime.
5. Find the minimum of  $x^4 + y^4 + z^4 - 4xyz$  over all real numbers  $x, y, z$ .
6. Al and Bo play the following game: there are 22 cards labeled 1 through 22. Al chooses one of them and places it on a table. Bo then places one of the remaining cards at the right of the one placed by Al such that the sum of the two numbers on the cards is a perfect square. Al then places one of the remaining cards such that the sum of the numbers on the last two cards played is a perfect square, and so on. The game ends when all the cards are played or no more cards can be placed on the table. The winner is the one who played the last card. Does Al have a winning strategy?
7. Find all triples  $(x, y, z)$  of positive integers such that  $x^3 + y^3 + z^3 = 2008$ .
8. In quadrilateral  $ABCD$  the point of intersection of the perpendicular bisectors of  $AD$  and  $BC$  lies on  $AB$ . Prove that  $AC = BD$  if and only if  $\angle A = \angle B$ .
9. In each square of an  $8 \times 8$  board Michael writes a positive integer not exceeding 10, such that each two numbers that appear in adjacent or diagonally adjacent squares are relatively prime. Prove that some number appears at least 11 times.
10. Let  $A_1A_2 \dots A_{10}$  be a regular decagon. Find the number of obtuse triangles whose vertices are among  $A_1, A_2, \dots, A_{10}$ .

# AwesomeMath Answer Sheet

Your Name

Problem Number

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Write neatly! Write all work inside the box. Do NOT write on the back of the page