



**AwesomeMath**  
making x,y,z as easy as a,b,c

## AwesomeMath Admission Test Cover Sheet

Your Name

Last Name

First Name

Admission Test

A

B

C

Check one

Contact Information

Phone Number

Please Print

Email

Number of pages (not including this cover sheet)

Awesome Math Test A  
January 13 - February 3, 2012

- Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.
- Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.
- All the work you present must be your own.
- Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.
- Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be emailed to [tandreescu@gmail.com](mailto:tandreescu@gmail.com) or mailed to the following address:

Dr. Titu Andreescu  
3425 Neiman Road, Plano TX 75025

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

# Test A

January 13 - February 3, 2012

1. With pennies, nickels, dimes, quarters, and half dollars, in how many ways can we make exact change for a dollar using precisely 21 coins?
2. Delete 20 digits from the number 123456789101112...9899100 to obtain the maximum possible remaining number.
3. Find all triples  $(x, y, z)$  of positive integers such that  $x^y + y^z + z^x = 1230$ .
4. Let  $P(x) = \frac{x^3}{3} - x^2 + x$ . Evaluate  $P(\sqrt[3]{3} + 1)$ .
5. Find the side-lengths  $a, b, c$  of a triangle satisfying the system of equations

$$\frac{abc}{-a + b + c} = 40, \frac{abc}{a - b + c} = 60, \frac{abc}{a + b - c} = 120.$$

6. Prove that  $64^{65} + 65^{64}$  is not a prime.
7. Find the maximum of  $3\sin^2 x + 8\sin x \cos x - 3\cos^2 x$ , where  $x \in \mathbb{R}$ .
8. Let  $a_n = n + \sqrt{n^2 - 1}$ ,  $n \geq 1$ . Prove that

$$\frac{1}{\sqrt{a_1}} + \frac{1}{\sqrt{a_2}} + \dots + \frac{1}{\sqrt{a_8}} = 2 + \sqrt{2}.$$

9. Solve in integers the system of equations

$$\begin{cases} xy - \frac{z}{3} = xyz + 1 \\ yz - \frac{x}{3} = xyz - 1 \\ zx - \frac{y}{3} = xyz - 9. \end{cases}$$

10. If  $a, b, c$  are the side-lengths of a triangle, prove that

$$\max(a, b, c) < \sqrt{\frac{2(a^2 + b^2 + c^2)}{3}}.$$

AwesomeMath Answer Sheet

Your Name

Problem Number

Page

Of

Write neatly! All work should be inside the box. Do NOT write on the back of the page!