AwesomeMath Academy Problem Solving 1 Readiness Guide

Introduction

This assessment is intended to help students decide if the Problem Solving 1 class is a good fit for them. The test is divided into two sections:

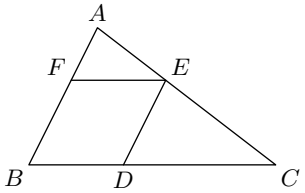
- **Prerequisite Skills** - These are to test basic skills the student will need as *prerequisites* for the topics to be covered in Problem Solving 1. Generally, to succeed in the class a student must be able to do basic computations (multiply, divide, etc.) without assistance. A student should also have some experience working with variables such as in solving linear equations. Students lacking in these skills will find it difficult to keep up with the rest of the class as we discuss problem solving strategies. We will also test a small amount of critical thinking here, as it is important that a student is ready to critically think upon entering the class.
- **Problem Solving** - These are to test the student's problem solving abilities. A student who completes 0-2 of these problems should find Problem Solving 1 to be the right difficulty for them. A student who completes 3 or more might consider looking at the Problem Solving 2 class instead.

Prerequisite Skills

1. Find the number x such that $4x + 9 = 3$
2. Find x and y if $3x + 4y = 19$ and $y = 2x$
3. The product of two integers is 270 and their difference is 3. What are the two numbers? Is there more than one possible answer?
4. A rectangle with integer side lengths has perimeter 24. What is the smallest possible area this rectangle could have?

Problem Solving

1. If $x + \frac{1}{x} = 1$, compute $x^3 + \frac{1}{x^3}$.
2. 10 kids are sitting in a row. Each kid receives 1, 2, or 3 candies (all candies are identical). How many ways are there to give candy to kids so that no 2 neighbors have exactly 4 candies total? (You may leave your answer in prime factorization)
3. Quadrilateral $BDEF$ is a rhombus with vertices on $\triangle ABC$. Given $AB = 10$ and $BC = 15$, find DE.



4. How many of the numbers, $100, 101, \ldots, 999$ have three different digits in increasing order or in decreasing order?
5. The hypotenuse of a right triangle is 10 inches and the radius of the inscribed circle is 1 inch. What is the perimeter of the triangle?
6. If n is a positive integer and D is a digit such that $\frac{n}{814} = 0.\overline{D75}$, what is the value of n?

Solutions

Prerequisite Skills

1. $x = -\frac{3}{2}$ or $x = -1.5$
2. $x = \frac{19}{11}, \ \ y = \frac{38}{11}$
3. Students might guess-and-check to find 15 and 18 are one possible answer. Critically thinking students might notice that -15 and -18 are also possible.
4. 11, achieved by the thinnest/longest possible rectangle

Problem Solving

1. Cubing both sides of the given equation yields $(x+\frac{1}{x})^3 = x^3+3x+\frac{3}{x}+\frac{1}{x^3} = x^3+\frac{1}{x^3}+3(x+\frac{1}{x}) = 1^3$. Substituting the given equation for $x+\frac{1}{x}$ yields $x^3+\frac{1}{x^3}+3(1) = 1$, thus
   $x^3+\frac{1}{x^3} = -2$
2. The first kid in line could get 1, 2, or 3 candies (3 options), each of the following kids will have 2 options regardless of what the previous kid received so the final answer is
   $3 * 2^9 = 1536$
3. Let s be the side length of the rhombus, then $DE = s$ and $BD = s$. Using similar triangles $\triangle CED$ and $\triangle CAB$ we find the s to be
   6
4. We can construct a number with increasing digits by just selecting three digits from 1, 2, 3, ..., 9 to form the number. Likewise a number with decreasing digits can be made by selecting three digits from 9, 8, 7, ..., 1, 0.
   $\binom{9}{3} + \binom{10}{3} = 204$
5. Let ABC be the triangle with $\angle C = 90°$, and the incircle touch the sides BC, AC, and AB at D, E, and F. Then the inradius 1 gives $CD = CE = 1$, let $BD = BF = x$ and $AF = AE = y$, then we find $x + y = AB = 10$, so the perimeter of the triangle is
   $AB + AC + BC = 10 + (x + 1) + (y + 1) = 22$
6. Express $0.\overline{D75}$ as $\frac{D75}{999}$, then the equation becomes $27n = 22(D75)$. This implies $D75$ is a multiple of 9 so $D + 7 + 5$ must be divisible by 9. The only possible digit is $D = 6$, which yields
   $n = 550$