

Explorer's Readiness Guide

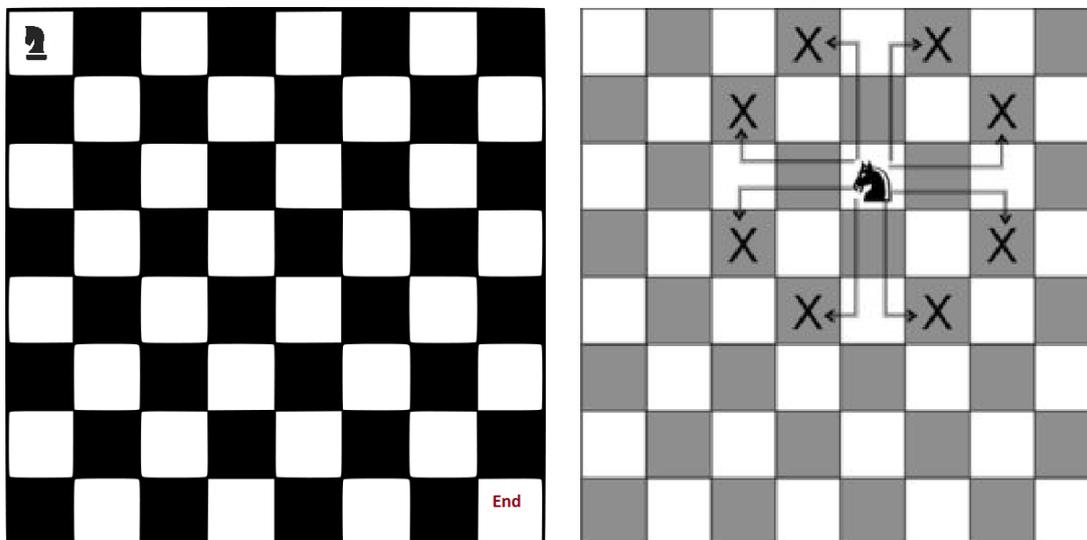
Introduction

The Explorer's in Problem Solving class is intended for kids who don't have a large breadth of mathematical background but love math and are interested in learning new kinds of math separate from school curriculum. As such, there aren't specific prerequisite math topics a student needs before being ready for the class. Instead, what is more important is a certain level of maturity. To elaborate:

- A student must be able to respectfully listen and take notes during the lesson without being a distraction to the other students.
- A student must have the personal drive to learn new techniques and tackle difficult problems which may take time and tenacity to solve.
- A student must be ready to think conceptually on certain topics and able to reason through why a pattern exists, not to accept that it simply exists.

The ability for rational and conceptual thought, especially to the “why” of a pattern or concept is possibly the most important but most difficult to gauge. Without this type of maturity a student might have all the ability to be in the class but not necessarily get anything out of it. On the following page is a logic puzzle with some questions meant to probe your child's ability to think through a pattern, some good answers follow on the page after but these may vary. It is advised that you give your child the puzzle and gauge their readiness for the class (in terms of logical maturity) through their responses. A child should be able to both write and explain their reasoning behind each answer.

A Riddle



In the game of chess, a knight moves in an L-shaped pattern (with each move it goes either two spaces in a horizontal direction and one in a vertical direction or two spaces in a vertical direction then one in a horizontal direction) as shown in the figure above on the right. Suppose a knight is placed on the top left of a chess board as shown above and on the left.

1. Why is it not possible to move the knight to the bottom right of the chessboard without ever moving up or to the left?
2. Why is it not possible to move the knight to the bottom right of the chessboard in an odd number of moves?
3. What is the least number of moves required to move the knight to the bottom right of the chessboard?¹

¹This problem and other interesting challenges for young mathematicians found in Math Leads for Mathletes <https://www.awesomemath.org/product/math-leads-for-mathletes/>

Solutions and Things to Watch For

1. Watch the way your child solves the problem. If you see them looking for paths at random, especially if they start repeating paths, that would show a lack of the mathematical maturity. A more mature child might try systematically exploring all possible paths (such as by taking as many down-heavy moves as possible, then taking one less than the maximum and seeing what happens from there, etc.). They might say at the end that it isn't possible because they've explored every path the knight could take without going up or to the left, but they should be able to explain how they know that every path has been explored. A very good observation for them to make would be "you will always stop two spaces away from the end," this is sign of pattern recognition and would be a good answer if they used that to justify there being no such path to the end. An exceptional child might give the perfect answer: that the knight must move a total of 14 spaces (7 to the right and 7 down) and each move takes it exactly 3 spaces closer, since 14 is not divisible by 3 the path is impossible.
2. Again, a child who tries various paths at random and says "it's not possible" after not finding one that works is lacking mathematical maturity. The desired answer is to notice that with each move the knight makes, the color of the square it is on changes. If it was on a white square before, it can only move to a black square, and vice versa. Since it must ultimately move from a white square to a white square it must necessarily take an even number of moves to do so. If a child cannot reason this through or does not notice the color-changing pattern, it is actually preferable to say "I don't know" rather than vigorously trying paths until it is deemed impossible. The child with mathematical maturity will realize there are too many paths to be able to try them all.
3. This is actually the least-probing question and therefore does not gauge the child's mathematical maturity as much as it does their problem solving skills. They should realize from part 1 that it must be greater than 4 and from part 2 that it cannot be 5. A good way of finding a path with 6 moves is to start with one of the 4-move paths they found earlier and add two moves that effectively move the knight the final two squares to the end. Once the path is found, you should ask the child "how do you know that is the least number of moves?" and they should be able to explain that they already knew from the previous two problems that it couldn't be less than six.