

AwesomeMath Academy

Problem Solving Level 3

Algebra: Find all positive integers a, b, c such that the equations

$$x^2 - ax + b = 0, \quad x^2 - bx + c = 0, \quad x^2 - cx + a = 0$$

have integer roots.

Number Theory: (St. Petersburg 1998) Let $d(n)$ denote the number of divisors of the natural number n . Prove that the sequence $d(n^2 + 1)$ does not become strictly increasing from any given point onward.

Combinatorics: Let n be a positive integer with $n > 2$. Fix $2n$ points in space in such a way that no four of them are in the same plane, and select any $n^2 + 1$ segments determined by the given points. Prove that these segments form at least one triangle.

Geometry: On the extension of chord KL of a circle centered at O , a point A is taken and tangents AP and AQ to the circle are drawn from it. Let M be the midpoint of PQ . Prove that $\angle MKO = \angle MLO$.