
AwesomeMath Admission Test - Sample

1. Is there an equiangular hexagon whose side lengths are (in some order) 2006, 2007, 2008, 2009, 2010, and 2011? (AwesomeMath Admission Test-B 2006)
2. Find the least positive integer n such that each n -element subset of the set $\{1, 2, \dots, 2007\}$ contains two elements, not necessarily distinct, such that their sum is a power of 2. (AwesomeMath Admission Test-A 2007)
3. There is a pile of 2009 chips on a table. You are allowed to perform repeatedly the following operation: choose a pile containing more than two chips, throw away a chip from the pile, then divide it into two smaller (not necessarily equal) piles. Is it possible that eventually all the piles on the table consist of exactly three chips? (AwesomeMath Admission Test-B 2009)
4. Find all integers n such that $n^2 + 2010n$ is a perfect square. (AwesomeMath Admission Test-B 2010)
5. Prove that the diameter of the incircle of a triangle ABC is equal to $\frac{1}{\sqrt{3}}(AB - BC + CA)$ if and only if $\angle A = 60^\circ$. (AwesomeMath Admission Test-C 2012)

6. If a, b, c are positive real numbers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{2013}{a + b + c}$, evaluate

$$\left(1 + \frac{a}{b}\right) \left(1 + \frac{b}{c}\right) \left(1 + \frac{c}{a}\right).$$

(AwesomeMath Admission Test-C 2013)

7. Find all integers n such that $n - 2014$ and $n + 2014$ are both triangular numbers. (AwesomeMath Admission Test-C 2014)
8. Find all functions $f : \mathbb{R}^* \rightarrow \mathbb{R}$ such that

$$f\left(\frac{2016}{x}\right) = 1 - xf(x), \text{ for all } x \in \mathbb{R}^*.$$

(AwesomeMath Admission Test-B 2016)

9. Let a, b, c be real numbers such that

$$(3a + 28b + 35c)(20a + 23b + 33c) = 1.$$

Prove that

$$a^2 + b^2 + c^2 > \frac{1}{2018}.$$

(AwesomeMath Admission Test-A 2018)

- 10.** An $n \times n$ magic square is filled with the numbers $1, 2, \dots, n^2$ such that the sum of the entries on each row, each column, and each of the two diagonals is the same. If, for some $n > 6$, we remove the number 37 from this square, the sum of all other entries in its row is 2019. Find n .
(AwesomeMath Admission Test-A 2019)