AwesomeMath Admission Test – Sample

The problems on this sample test are provided so that students can gain an understanding of the mathematical topics and levels of difficulty involved. **Part I** is for students interested in taking Level 1 and Level 2 classes. **Part II** is for students interested in taking Level 3 and Level 4 classes. Do not be discouraged if you cannot solve all questions. We want to see the solutions you come up with no matter how many problems you solve.

Part I

Levels 1 & 2

1. What is the greatest integer n for which $n^2 + 2009n$ is the square of an integer?

2. In the table

0						
1	2	3				
4	5	6	7	8		
9	10	11	12	13	14	15

what is the number directly below 2010?

3. What is the product of the real zeros of the polynomial

$$p(x) = x^4 + 4x^3 + 6x^2 + 4x - 2011?$$

4. The sum of some consecutive integers is 2012. Find the smallest of these integers.

5. Find all pairs (m, n) of positive integers such that

$$m(n+1) + n(m-1) = 2013.$$

6. Find all four-digit numbers n whose sum of digits is equal to $2014 - n$.

7. Find the least positive integer that is divisible by precisely 2015 perfect squares.

8. Solve in positive integers the equation $x^2y^2z^2 - \min(x^2, y^2, z^2) = 2016$.

9. Let a be a positive real number such that $a^2 + \dfrac{16}{a^2} = 2017$. Evaluate

$$\sqrt{a} + \frac{2}{\sqrt{a}}.$$

10. In the addition $\overline{AWE} + \overline{SOME} = 2018$, each letter represents a different nonzero digit of the decimal system. Find the minimum possible value of $W \cdot M$.

Part II

Levels 3 & 4

1. Let $P(x) = 2009x^9 + a_1x^8 + \cdots + a_9$ such that

$$P\left(\frac{1}{n}\right) = \frac{1}{n^3}, \ n = 1, 2, \ldots, 9.$$

Find $P\left(\frac{1}{10}\right)$.

2. In quadrilateral $ABCD$, $\angle B = \angle C = 120^\circ$ and

$$AD^2 = AB^2 + BC^2 + CD^2.$$

Prove that $ABCD$ has an inscribed circle.

3. Evaluate

$$\left(1 - \frac{2011}{2}\right)\left(1 - \frac{2011}{3}\right)\cdots\left(1 - \frac{2011}{2010}\right).$$

4. For a positive integer N, let $r(N)$ be the number obtained by reversing the digits of N. For example, $r(2013) = 3102$. Find all 3-digit numbers N for which $r^2(N) - N^2$ is the cube of a positive integer.

5. If a, b, c are positive real numbers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{2013}{a+b+c}$, evaluate

$$\left(1 + \frac{a}{b}\right)\left(1 + \frac{b}{c}\right)\left(1 + \frac{c}{a}\right).$$

6. Find all integers n such that $n - 2014$ and $n + 2014$ are both triangular numbers.

7. Let $P(x) = x^{2015} + x + 5$. Find the remainder when $P(x)$ is divided by $x^5 - x$.

8. All but one of the squares of an $n \times n$ chessboard are labeled with numbers from the set $\{8, 16, \ldots, 8n^2\}$, none of which being used more than once, such that the sum of the numbers on each row and each column is 2016. Find the number that has not been used.

9. Find the least real number a such that

$$x^4 + ax^3 + 2017x^2 - 360x + 16 \geq 0$$

for all positive real numbers x.

10. Let a, b, c be real numbers such that

$$(3a + 28b + 35c)(20a + 23b + 33c) = 1.$$

Prove that

$$a^2 + b^2 + c^2 > \frac{1}{2018}.$$