**Junior Problems**

**J619.** In triangle $ABC$,

$$AB^4 + BC^4 + CA^4 = 2AB^2 \cdot BC^2 + AB^2 \cdot CA^2 + 2BC^2 \cdot CA^2.$$ 

Find all possible values of $\angle A$.

*Proposed by Adrian Andreescu, Dallas, USA*

**J620.** Let $ABC$ be a right triangle and let $M$ be the midpoint of the hypotenuses $BC$. It is known that $AM^2 = AB \cdot AC$. Find the measure of angle $ACB$.

*Proposed by Vasile Lupulescu, Târgu Jiu, România*

**J621.** Let $ABC$ be a triangle with $AB \neq AC$ and let $I$ be its incenter. Let $X$ be the midpoint of segment $BC$. Line $XI$ intersects the altitude from $A$ in $Y$. Prove that $AY = r$.

*Proposed by Mihaela Berindeanu, Bucharest, România*

**J622.** Let $a, b, c$ be real numbers such that $a, b, c \in \left[ \frac{1}{2}, 1 \right]$. Prove that

$$\frac{a}{\sqrt{b} + \sqrt{c}} + \frac{b}{\sqrt{c} + \sqrt{a}} + \frac{c}{\sqrt{a} + \sqrt{b}} < 2.$$ 

*Proposed by Nguyen Viet Hung, Hanoi University of Science, Vietnam*

**J623.** Let $m_a, m_b, m_c$ be the medians in a triangle $ABC$. Prove that

$$\frac{m_a^4}{m_b + m_c - m_a} + \frac{m_b^4}{m_c + m_a - m_b} + \frac{m_c^4}{m_a + m_b - m_c} \geq m_a^3 + m_b^3 + m_c^3.$$ 

*Proposed by Mihaly Bencze, Brașov and Neculai Stanciu, Buzău, România*

**J624.** In triangle $ABC$ let $M, N, P$ be the midpoints of $BC, CA, AB$, respectively, and let $D, E, F$ be the feet of the altitudes on sides $BC, CA, AB$, respectively. Prove that

$$\frac{DM + EN + FP}{2} \geq \max \{m_a, m_b, m_c\} - \min \{m_a, m_b, m_c\}.$$ 

*Proposed by Marius Stânean, Zalău, România*
Senior Problems

S619. Let \(a, b, c \in [0, 1]\), no two of which are zero. Prove that
\[
\frac{ab + 1}{a + b} + \frac{bc + 1}{b + c} + \frac{ca + 1}{c + a} \geq \frac{ab + bc + ca + 3}{a + b + c} + 1.
\]

Proposed by Nguyen Viet Hung, Hanoi University of Science, Vietnam

S620. Let \(a, b, c, d\) be positive real numbers. Prove that
\[
(abc + abd + acd + bcd)^2 \geq 4abcd(ab + bc + cd + da).
\]

Proposed by An Zhenping, Xianyang Normal University, China

S621. Find all positive integers \(n\) for which there are positive integers \(a\) and \(b\) and a non-degenerate triangle with side lengths \(n, 3a, 5b\).

Proposed by Josef Tkadlec, Czech Republic

S622. Let \(ABC\) be a triangle inscribed in a circle \(\Gamma\) of center \(O\). The tangents at \(A\) and \(C\) to \(\Gamma\) intersect each other in \(P\). The line \(BP\) intersect \(\Gamma\) in \(Q\) and let \(S\) be the midpoint of \(BQ\). Prove that \(\angle ACQ = \angle BCS\).

Proposed by Mihaela Berindeanu, Bucharest, Romania

S623. Let \(a_1, a_2, \ldots, a_n\) and \(b_1, b_2, \ldots, b_n\) \((n \geq 2)\) be positive real numbers satisfying
\[
\frac{a_1}{b_1} \geq \frac{a_2}{b_2} \geq \ldots \geq \frac{a_n}{b_n}.
\]
Prove that
\[
\sqrt{a_1} + \sqrt{b_1} + \sqrt{a_2} + \sqrt{b_2} + \ldots + \sqrt{b_{n-1}} + \sqrt{a_n} + \sqrt{b_n} > \sqrt{a_1 + b_1} + \sqrt{a_2 + b_2} + \ldots + \sqrt{a_n + b_n}.
\]

Proposed by Waldemar Pompe, Warsaw, Poland

S624. Prove that the following inequality holds for all positive real numbers \(a, b, c\):
\[
\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq \sqrt{\frac{b + c}{2a}} + \sqrt{\frac{c + a}{2b}} + \sqrt{\frac{a + b}{2c}}.
\]

Proposed by Nguyen Viet Hung, Hanoi University of Science, Vietnam
Undergraduate Problems

**U619.** Find all polynomials $P(x)$ with real coefficients such that
\[ P(x)(P(x) - 2P(y))^2 + (2P(x) - P(y))^2P(y) = P(xP(x)) + P(yP(y)), \]
for all $x, y \in \mathbb{R}$.

*Proposed by Titu Andreescu, University of Texas at Dallas, USA*

**U620.** Evaluate
\[ \lim_{n \to \infty} \sum_{k=1}^{n} \frac{k^2}{2k^2 - 2nk + n^2}. \]

*Proposed by Vasile Lupulescu, Târgu Jiu, România*

**U621.** Let $x, y, z$ be nonnegative real numbers such that $x + y + z = 2$. Find the minimum of
\[ \sqrt{4 + 2x^2} + \sqrt{54 - 36\sqrt{2} + 4y^2} + \sqrt{8 + 2z^2}. \]

*Proposed by Paolo Perfetti, Universita degli studi di Tor Vergata, Roma, Italy*

**U622.** Prove that in any acute triangle $ABC$,
\[ \left( \frac{4S}{3R} \right)^4 \geq \frac{3(a^2 + b^2 - c^2)(b^2 + c^2 - a^2)(c^2 + a^2 - b^2)}{a^2 + b^2 + c^2}. \]

*Proposed by Marius Stânean, Zalău, România*

**U623.** Find all positive real numbers $a$ for which the sequence
\[ x_n = \frac{\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}}{n^a} \]
converges and find its limits in those cases.

*Proposed by Mircea Becheanu, Canada*

**U624.** Let $p$ be a prime number. For every positive integer $n$ denote by $\text{rad}_p(n)$ the product of all prime divisors of $n$, except $p$. Let $f : \mathbb{N} \to \mathbb{N}$ be a multiplicative function for which there is a nonzero integer $c$ such that
\[ \text{rad}_p(n)|f(n + 1) - c. \]

Prove that $f(n) = n^r$, for some positive integer $r$.

*Proposed by Navid Safaei, Sharif University of Technology, Tehran, Iran*
O619. Let \( l \) be a nonnegative integer. Prove that there are infinitely many positive integers \( k \geq l \), for each of which there are infinitely many blocks of \( k \) consecutive positive integers such that every such block contains precisely \( l \) numbers that can be represented as the sum of two perfect squares of integers.

Proposed by Titu Andreescu, USA and Marian Tetiva, România

O620. Prove that for any positive integer \( n \) there is at most one triplet of positive integers \( a \leq b \leq c \) such that 
\[(a + b)(b + c)(c + a)(a + b + c + n)\]
is a power of a prime.

Proposed by Josef Tkadlec, Czech Republic and Ján Mazák, Slovakia

O621. Let \( a_1, \ldots, a_k \) and \( b_1, \ldots, b_k \) be sets of integers, with \( a_1, \ldots, a_k \) positive and mutually distinct, and let \( \varepsilon \) be a positive real number. Prove that there are infinitely many positive integers \( n \) such that 
\[(a_1 n + b_1) \cdots (a_k n + b_k)\]
divides \( \lceil \varepsilon n \rceil ! \). (As usual, \( \lfloor x \rfloor \) denotes the integer part of the real number \( x \).)

Proposed by Titu Andreescu, USA and Marian Tetiva, România

O622. Determine all positive integers \( n \) for which the numbers \( 1, 2, \ldots, n \) can be written on a paper in such an order that for each \( k = 1, 2, \ldots, n \) the sum of the first \( k \) numbers is a multiple of \( k \).

Proposed by Josef Tkadlec, Czech Republic

O623. Prove that there is a positive integer \( n \) and a list of bases \( b_1, b_2, \ldots, b_{2022} \) such that \( n \) is a 2023-palyndrome in each of the bases \( b_1, b_2, \ldots, b_{2022} \).

Proposed by Navid Safaei, Sharif University of Technology, Tehran, Iran

O624. Let \( ABCD \) be a convex quadrilateral with \( \angle BCA = \angle DCA \). Let \( r_1 \) and \( r_2 \) be the inradii of triangles \( ABC \) and \( ACD \), respectively. Let \( r_3 \) be the radius of a circle that passes through \( C \) and is tangent to rays \( AC \) and \( AB \). Similarly, let \( r_4 \) be the radius of a circle that passes through \( C \) and is tangent to rays \( AC \) and \( AD \). Prove that
\[
\frac{1}{r_1} + \frac{1}{r_4} = \frac{1}{r_2} + \frac{1}{r_3}.
\]

Proposed by Waldemar Pompe, Warsaw, Poland