

CleverMath: Geometry and Trigonometry

Think deeper – one challenge at a time

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Preface

Many of the problems in this book were originally featured in the online CleverMath program, offered by AwesomeMath in 2015. This program was designed to provide students with high-quality math training through carefully selected problems that would challenge their problem-solving abilities and deepen their understanding of key mathematics concepts. Over the years, these problems have been refined and compiled into this book to create a structured and comprehensive learning resource.

This book is part of a five-volume series, with the other volumes focusing on combinatorics, algebra, number theory, and calculus. Each volume is dedicated to a fundamental branch of mathematics, providing learners with a comprehensive foundation in problem-solving techniques essential for mathematics competitions and beyond.

All problems presented have a one to three-digit answer, making them ideal preparation for the AIME (American Invitational Mathematics Examination). Nevertheless, the problem set is diverse, covering a wide range of difficulty levels – some questions align with the AMC (American Mathematics Competitions) 10/12 while others are more advanced, making them suitable for Olympiad-level training.

Each problem is accompanied by a detailed and rigorous solution, ensuring that students not only arrive at the correct answer but also understand the underlying reasoning. Many solutions go beyond standard explanations, offering insightful remarks, alternative approaches, and connections to broader mathematics concepts. These additional observations encourage deeper thinking, allowing students to develop flexibility in their problem-solving strategies.

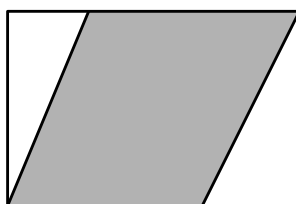
This book is tailored to students preparing for math competitions, their coaches, or anyone with a passion for problem-solving. With a carefully curated collection of challenging and thought-provoking problems, students will refine their analytical skills, hone their problem-solving techniques, and gain confidence in tackling even more complex mathematics problems.

Titu Andreescu

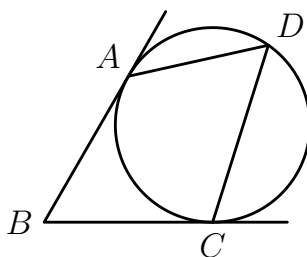
Problems

Problem 1. In a convex n -gon, the average of the angles is 162° . Find n .

Problem 2. A rhombus is inscribed in a 7×9 rectangle, as shown below. Then the side-length of the rhombus can be expressed in the form m/n , where m and n are relatively prime positive integers. Find $m + n$.



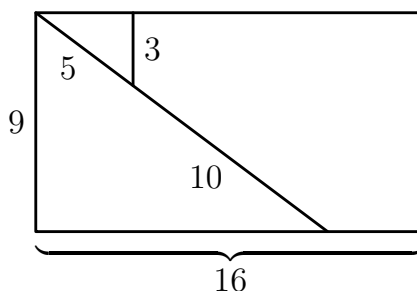
Problem 3. In the diagram below, $\angle ABC = 48^\circ$. Find $\angle ADC$, in degrees.



Problem 4. In a triangle with side-lengths a , b , and c , $(a + b + c)(a + b - c) = 3ab$. Find the angle opposite the side with length c , in degrees.

Problem 5. The medians of triangle ABC intersect at point G . Given that $\angle AGB = 90^\circ$ and $AB = 20$. Find the length of the median corresponding to vertex C .

Problem 6. A 9×16 rectangle is dissected into three pieces, as shown below, with the lengths indicated. The pieces can be assembled to form a square. Find the side-length of the square.



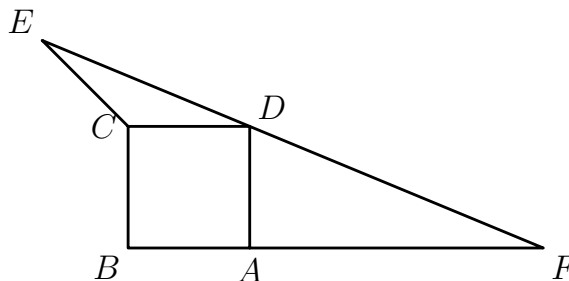
Problem 7. Let d be the length of a space diagonal (going from one vertex to the opposite vertex) of a cube. Then the surface area of the cube can be expressed in the form ad^b , where a and b are positive integers. Find $a + b$.

Problem 8. A rectangle is divided into four smaller rectangles. The smaller rectangles, along with their areas, are shown below. What is the area of the fourth rectangle?

459	306
?	72

Problem 9. In triangle ABC , $\angle C = 90^\circ$, $AB = 20$, and $AC = 12$. Let D be a point on BC such that $AD = BD + 8$. Find CD .

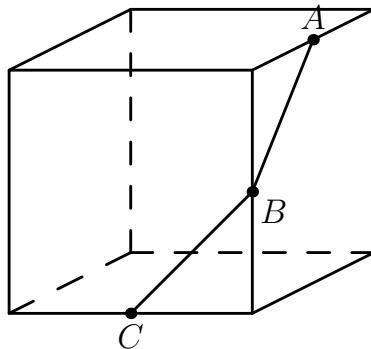
Problem 10. Let $ABCD$ be a square. Let E be a point such that $CE = AB$, and let F be the intersection of DE and AB . If $DE = 14$ and $DF = 28$, then the length of BF can be expressed in the form $a\sqrt{b}$, where a and b are positive integers, and b is not divisible by the square of a prime. Find $a + b$.



Problem 11. Sam has three spheres made of solid gold, with radii 11, 15, and 27. He melts them down, and casts them to form one large sphere. Find the radius of this sphere.

Problem 12. In triangle ABC , $\angle C = 90^\circ$, $AB = 10$, and $AC = 2BC$. Find the area of triangle ABC .

Problem 13. Let A , B , and C be the midpoints of the sides of a cube, as shown below. Find $\angle ABC$, in degrees.



Problem 14. In triangle ABC , $AB = 20$, $AC = 15$, and $BC = 7$. Line segment BC is extended past C to D such that triangles DAB and DCA are similar. Find DC .

Problem 15. The angles of a quadrilateral A, B, C, D form an arithmetic sequence, in that order. If $A = 57^\circ$, then find C , in degrees.

Problem 16. In the coordinate plane, let $A = (-2, 3)$, $B = (b_1, b_2)$, $C = (11, 1)$, and $D = (d_1, d_2)$. If quadrilateral $ABCD$ is a rectangle of area 60, then find $b_1 + b_2 + d_1 + d_2$.

Problem 17. Let $ABCD$ be a rectangle. Let P be a point on side AB such that $AP : PB = 4/5$. Let Q be a point on side CD such that $CQ : QD = 11 : 3$. Let R be the intersection of AC and PQ . Then $AR : AC$ can be expressed in the form m/n , where m and n are relatively prime positive integers. Find $m + n$.

Problem 18. Three interior angles of an polygon with n sides are 143° , 243° , and 343° . Find the smallest possible value of n .

Problem 19. Let $ABCD$ be a convex quadrilateral such that $\angle BAC = 20^\circ$, $\angle ACB = \angle BDC = 40^\circ$, and $\angle ADB = 80^\circ$. Find $\angle ABD$, in degrees.

Problem 20. Two perpendicular chords AB and CD of a circle intersect at the center O . A second circle is tangent to the first circle at B , and intersects AO and CO at P and Q , respectively. If $AP = 10$ and $CQ = 7$, then the radius of the larger circle can be expressed in the form m/n , where m and n are relatively prime positive integers. Find $m + n$.